10-3 Areas of Regular Polygons

Vocabulary

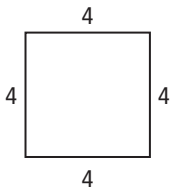
Review

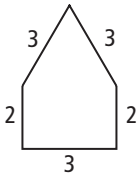
Write T for *true* or F for *false*.

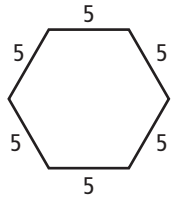
_____ **1.** In a *regular polygon*, all sides are congruent.

_____ **2.** In a *regular polygon*, all angles are acute.

3. Cross out the figure that is NOT a *regular polygon*.







Vocabulary Builder

apothem (noun) AP **uh them**

Related Words: center, regular polygon

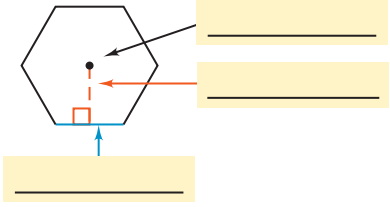
Definition: The **apothem** is the perpendicular distance from the center of a regular polygon to one of its sides.

Use Your Vocabulary

4. Underline the correct word to complete the statement.

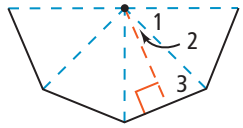
In a regular polygon, the *apothem* is the perpendicular distance from the center to a(n) angle / side .

5. Label the regular polygon below using *apothem, center,* or *side*.



Problem 1 Finding Angle Measures

Got It? **At the right, a portion of a regular octagon has radii and an apothem drawn. What is the measure of each numbered angle?**



6. A regular octagon has ____ sides.

7. Circle the type of triangles formed by the radii of the regular octagon.

equilateral isosceles right

8. Use the justifications below to find the measure of each numbered angle.

$m\angle 1 = \frac{360}{\quad} =$ ____ Divide 360 by the number of sides.

$m\angle 2 =$ ____ $(m\angle 1)$ The apothem bisects the vertex angle of the triangle formed by the radii.

$= \frac{1}{2}($ ____ $) =$ ____

$90 + m\angle 2 + m\angle 3 =$ ____ Triangle Angle-Sum Theorem

$90 +$ ____ $+ m\angle 3 =$ ____ Substitute.

____ $+ m\angle 3 =$ ____ Simplify.

$m\angle 3 =$ ____ Subtraction Property of Equality

9. Write the measure of each numbered angle.

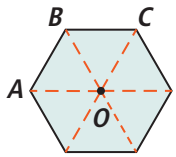
$m\angle 1 =$ ____ $m\angle 2 =$ ____ $m\angle 3 =$ ____

take note

Postulate 10-1 and Theorem 10-6

Postulate 10-1 If two figures are congruent, then their areas are equal.

The isosceles triangles in the regular hexagon at the right are congruent. Complete each statement.

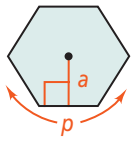


10. If the area of $\triangle AOB$ is 24 in.^{2}, then the area of $\triangle BOC$ is ____ in.^{2}.

11. If the area of $\triangle BOC$ is 8 cm^2, then the area of $\triangle AOC$ is ____ cm^2.

Theorem 10-6 Area of a Regular Polygon

The area of a regular polygon is half the product of the apothem and the perimeter.



$$A = \frac{1}{2}ap$$

Complete.

12. apothem: 10 perimeter: 80 area: $\frac{1}{2}(10) \cdot$ ____

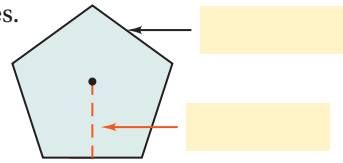
13. apothem: 5 perimeter: $30\sqrt{3}$ area: $\frac{1}{2} \cdot$ ____ $\cdot$ ____

14. apothem: $5\sqrt{3}$ perimeter: 60 area: ____ $\cdot$ ____ $\cdot$ ____

Problem 2 Finding the Area of a Regular Polygon

Got It? **What is the area of a regular pentagon with an 8-cm apothem and 11.6-cm sides?**

15. Label the regular pentagon with the lengths of the apothem and the sides.



16. Use the justifications below to find the perimeter.

$p = ns$ Use the formula for the perimeter of an n-gon.

$= \square(11.6)$ Substitute for n and for s.

$= \square$ Simplify.

17. Use the justifications below to find the area.

$A = \frac{1}{2}ap$ Use the formula for the area of a regular polygon.

$= \frac{1}{2} \cdot \square \cdot \square$ Substitute for a and for p.

$= \square$ Simplify.

18. The regular pentagon has an area of $\square$ cm^2.

Problem 3 Using Special Triangles to Find Area

Got It? **The side of a regular hexagon is 16 ft. What is the area of the hexagon? Round your answer to the nearest square foot.**

19. Use the information in the problem to complete the problem-solving model below.

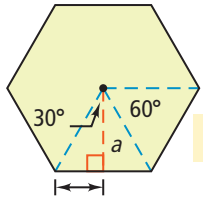
Know
I know that the length of each side of the regular hexagon is $\square$ ft.

Need

Plan
Draw a diagram to help find the length of the apothem.

Then use the perimeter and area formulas.

Use the diagram at the right.



20. Label the diagram.

21. Circle the relationship you can use to find the length of the apothem.

hypotenuse $= 2 \cdot$ shorter leg longer leg $= \sqrt{3} \cdot$ shorter leg

22. Complete.

length of shorter leg $=$ $\square$ ft

length of longer leg (apothem) $=$ $\square$ ft

23. Use the formula $p = ns$ to find the perimeter of the hexagon.

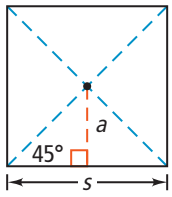
24. Now use the perimeter and the formula $A = \frac{1}{2}ap$ to find the area of the hexagon.

25. To the nearest square foot, the area of the hexagon is _____ ft^2.

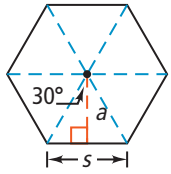
Lesson Check • Do you UNDERSTAND?

What is the relationship between the side length and the apothem in each figure?

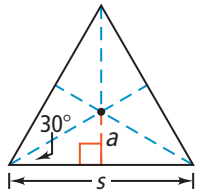
square



regular hexagon



equilateral triangle



26. The radius and apothem form what type of triangle in each figure?

square

_____° - _____° - _____° triangle

regular hexagon

_____° - _____° - _____° triangle

equilateral triangle

_____° - _____° - _____° triangle

27. Complete to show the relationship between the side length and the apothem.

square

leg = leg

$a =$ _____ s

regular hexagon

longer leg = $\sqrt{3}$ · shorter leg

$a = \sqrt{3} \cdot$ _____ s

$a =$ _____ s

equilateral triangle

longer leg = $\sqrt{3}$ · shorter leg

$\frac{1}{2}s = \sqrt{3} \cdot$ _____

$s =$ _____ a

Math Success

Check off the vocabulary words that you understand.

☐ radius of a regular polygon ☐ apothem

Rate how well you can *find the area of a regular polygon.*

Need to review 0 2 4 6 8 10 Now I get it!