

# 4-6

## Congruence in Right Triangles



### Vocabulary

#### Review

Write T for *true* or F for *false*.

- Segments that are *congruent* have the same length.
- Polygons that are *congruent* have the same shape but are not always the same size.
- In *congruent* figures, corresponding angles have the same measure.

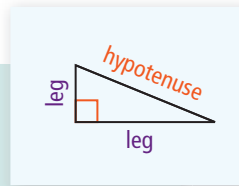
#### Vocabulary Builder

**hypotenuse** (noun) hy PAH tuh noos

**Related Word:** leg

**Definition:** The **hypotenuse** is the side opposite the right angle in a right triangle.

**Main Idea:** The **hypotenuse** is the longest side in a right triangle.



#### Use Your Vocabulary

Underline the correct word(s) to complete each sentence.

- One side of a right triangle is / is not a *hypotenuse*.
- A right triangle has one / two / three *legs*.
- The length of the *hypotenuse* is always equal to / greater than / less than the lengths of the *legs*.

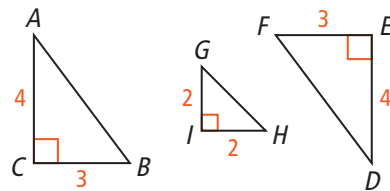
Use the triangles at the right for Exercises 7 and 8.

- Cross out the side that is NOT a *hypotenuse*.

$\overline{BC}$        $\overline{AB}$        $\overline{GH}$        $\overline{FD}$

- Circle the *leg(s)*.

$\overline{AC}$        $\overline{AB}$        $\overline{HI}$        $\overline{ED}$



You can prove that two triangles are congruent without having to show that *all* corresponding parts are congruent. In this lesson, you will prove right triangles congruent by using one pair of right angles, a pair of hypotenuses, and a pair of legs.

**Take note**

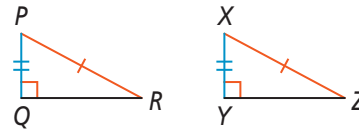
**Theorem 4-6 Hypotenuse-Leg (HL) Theorem and Conditions**

**Theorem**

If the hypotenuse and a leg of one right triangle are congruent to the hypotenuse and leg of another right triangle, then the triangles are congruent.

**If . . .**

$\triangle PQR$  and  $\triangle XYZ$  are right triangles,  
 $\overline{PR} \cong \overline{XZ}$ , and  $\overline{PQ} \cong \overline{XY}$



**Then . . .**

$\triangle PQR \cong \triangle XYZ$

9. To use the HL Theorem, the triangles must meet three conditions. Complete each sentence with *right* or *congruent*.

There are two   ?   triangles.

---



---

The triangles have   ?   hypotenuses.

---



---

There is one pair of   ?   legs.

---



---

**Use the information in the Take Note for Exercises 10–12.**

10. How do the triangles in the Take Note meet the first condition in Exercise 9? Explain.

---



---

11. How do the triangles in the Take Note meet the second condition in Exercise 9? Explain.

---



---

12. How do the triangles in the Take Note meet the third condition in Exercise 9? Explain.

---

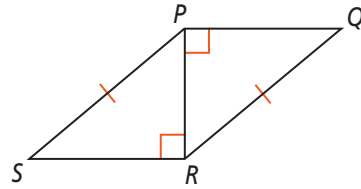


---



### Problem 1 Using the HL Theorem

**Got It?** Given:  $\angle PRS$  and  $\angle RPQ$  are right angles,  $\overline{SP} \cong \overline{QR}$   
 Prove:  $\triangle PRS \cong \triangle RPQ$



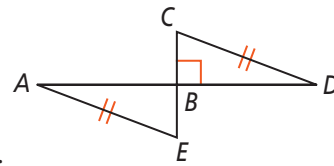
13. Complete each step of the proof.

Given $\angle PRS$ and $\angle$ <span style="background-color: yellow; border: 1px solid black; display: inline-block; width: 20px; height: 15px;"></span> are right angles.	Given $\overline{SP} \cong$ <span style="background-color: yellow; border: 1px solid black; display: inline-block; width: 20px; height: 15px;"></span>	Reflexive Prop. of $\cong$ $\overline{PR} \cong$ <span style="background-color: yellow; border: 1px solid black; display: inline-block; width: 20px; height: 15px;"></span>
↓	↓	↓
Definition of right triangle $\triangle PRS$ and $\triangle$ <span style="background-color: yellow; border: 1px solid black; display: inline-block; width: 20px; height: 15px;"></span> are right triangles.	HL Theorem $\triangle PRS \cong \triangle$ <span style="background-color: yellow; border: 1px solid black; display: inline-block; width: 20px; height: 15px;"></span>	



### Problem 2 Writing a Proof Using the HL Theorem

**Got It?** Given:  $\overline{CD} \cong \overline{EA}$ ,  $\overline{AD}$  is the perpendicular bisector of  $\overline{CE}$   
 Prove:  $\triangle CBD \cong \triangle EBA$



14. Circle what you know because  $\overline{AD}$  is the perpendicular bisector of  $\overline{CE}$ .

$\angle CBD$ and $\angle EBA$ are right angles.	$\angle CBD$ and $\angle EBA$ are acute angles.
$B$ is the midpoint of $\overline{AD}$ .	$B$ is the midpoint of $\overline{CE}$ .

15. Circle the congruent legs.

$\overline{AB}$	$\overline{CB}$	$\overline{DB}$	$\overline{EB}$
-----------------	-----------------	-----------------	-----------------

16. Write the hypotenuse of each triangle.

$\triangle CBD$ <span style="background-color: yellow; border: 1px solid black; display: inline-block; width: 20px; height: 15px;"></span>	$\triangle EBA$ <span style="background-color: yellow; border: 1px solid black; display: inline-block; width: 20px; height: 15px;"></span>
--	--

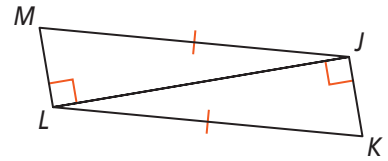
17. Complete the proof.

Statements	Reasons
1) $\overline{CD} \cong$ <span style="background-color: yellow; border: 1px solid black; display: inline-block; width: 20px; height: 15px;"></span>	1) Given
2) $\angle CBD$ and $\angle$ <span style="background-color: yellow; border: 1px solid black; display: inline-block; width: 20px; height: 15px;"></span> are right $\angle$ s.	2) Definition of $\perp$ bisector
3) $\triangle CBD$ and $\triangle$ <span style="background-color: yellow; border: 1px solid black; display: inline-block; width: 20px; height: 15px;"></span> are right $\triangle$ s.	3) Definition of right $\triangle$
4) $\overline{CB} \cong$ <span style="background-color: yellow; border: 1px solid black; display: inline-block; width: 20px; height: 15px;"></span>	4) Definition of $\perp$ bisector
5) $\triangle CBD \cong$ <span style="background-color: yellow; border: 1px solid black; display: inline-block; width: 20px; height: 15px;"></span>	5) HL Theorem



## Lesson Check • Do you UNDERSTAND?

**Error Analysis** Your classmate says that there is not enough information to determine whether the two triangles at the right are congruent. Is your classmate correct? Explain.



Write T for *true* or F for *false*.

18. There are three right angles.
19. There are two right triangles.
20. There are two congruent hypotenuses.
21. There are no congruent legs.
22. You need to use the Reflexive Property of Congruence.
23.  $\overline{LJ} \cong \overline{LJ}$  is given.
24. Do you always need three congruent corresponding parts to prove triangles congruent? Explain.

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

25. Is your classmate correct? Explain.

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_



## Math Success

Check off the vocabulary words that you understand.

- hypotenuse                       legs of a right triangle

Rate how well you can use *the Hypotenuse-Leg (HL) Theorem*.

