

5-6

Inequalities in One Triangle



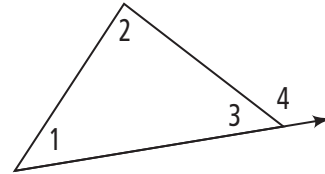
Vocabulary

Review

- Circle the labeled *exterior angle*.
- Write the *Exterior Angle Theorem* as it relates to the diagram.

$$m\angle \square = m\angle \square + m\angle \square$$

- Draw an *exterior angle* adjacent to $\angle 1$ and label it $\angle 5$.



Circle the statement that represents an *inequality* in each pair below.

- $x \neq 32$
 $x = 32$

- The number of votes is equal to 10,000.
The number of votes is greater than 10,000.

Complete each statement with an inequality symbol.

- y is less than or equal to z .
 y \square z
- The temperature t is at least 80 degrees.
 t \square 80°

Vocabulary Builder

compare (verb) kum PEHR

Other Word Form: comparison (noun)

Definition: To **compare** is to examine two or more items, noting similarities and differences.

Math Usage: Use inequalities to **compare** amounts.

There are more letters in the word *comparison* than in the word *compare*.

Use Your Vocabulary

- Complete each statement with the appropriate form of the word *compare*.

NOUN By ?, a spider has more legs than a beetle.

VERB You can ? products before deciding which to buy.

VERB To ? quantities, you can write an equation or an inequality.

Take note

Property Comparison Property of Inequality

If $a = b + c$ and $c > 0$, then $a > b$.

9. Circle the group of values that satisfies the Comparison Property of Inequality.

$a = 5, b = 5, \text{ and } c = 0$

$a = 5, b = 2, \text{ and } c = 3$

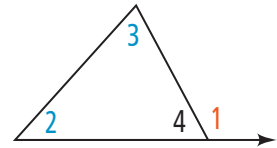
$a = 8, b = 6, \text{ and } c = 1$

Take note

Corollary Corollary to the Triangle Exterior Angle Theorem

The measure of an **exterior angle** of a triangle is greater than the measure of each of its **remote interior angles**.

10. Circle the angles whose measures are always less than the measure of $\angle 1$.



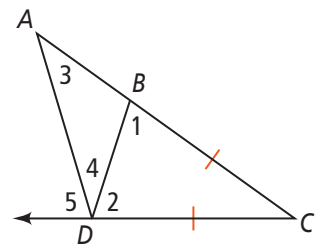
Problem 1 Applying the Corollary

Got It? Use the figure at the right. Why is $m\angle 5 > m\angle C$?

Write the justification for each statement.

11. $\angle 5$ is an exterior angle of $\triangle ADC$.

12. $m\angle 5 > m\angle C$



You can use the Corollary to the Triangle Exterior Angle Theorem to prove the following theorem.

Take note

Theorem 5-10 and Theorem 5-11

Theorem 5-10

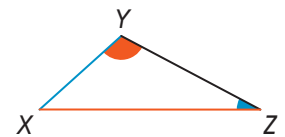
If two sides of a triangle are not congruent, then the **larger angle** lies opposite the **longer side**.

If $XZ > XY$, then $m\angle Y > m\angle Z$.

13. Theorem 5-11 is related to Theorem 5-10. Write the text of Theorem 5-11 by exchanging the words “larger angle” and “longer side.”

Theorem 5-11 If two sides of a triangle are not congruent, then

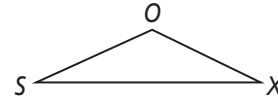
_____.





Problem 3 Using Theorem 5-11

Got It? Reasoning In the figure at the right, $m\angle S = 24$ and $m\angle O = 130$. Which side of $\triangle SOX$ is the shortest side?



Explain your reasoning.

14. By the Triangle Angle-Sum Theorem, $m\angle S + m\angle O + m\angle X = 180$,
so $m\angle X = \square - m\angle S - m\angle O$.

15. Use the given angle measures and the equation you wrote in Exercise 14 to find $m\angle X$.

$$m\angle X = \square - \square - \square = \square$$

16. Complete the table below.

angle	\square	\square	\square
angle measure	130	26	24
opposite side	\square	\square	\square

17. Which is the shortest side? Explain.

The shortest side is \square because it is opposite the smallest angle, $\angle \square$.

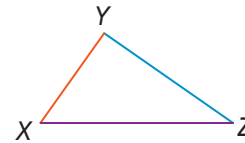
take note

Theorem 5-12 Triangle Inequality Theorem

The sum of the lengths of any two sides of a triangle is greater than the length of the third side.

18. Complete each inequality.

$$XY + YZ > \square \quad YZ + ZX > \square \quad ZX + XY > \square$$



Problem 4 Using the Triangle Inequality Theorem

Got It? Can a triangle have sides with lengths 2 m, 6 m, and 9 m? Explain.

19. Complete the reasoning model below.

Think	Write
The sum of the lengths of any two sides must be greater than the length of the third side.	$2 + 6 = 8$ $6 + 9 = 15$ $2 + 9 = 11$
I need to write three sums and three inequalities.	$8 \square 9$ $15 \square 2$ $11 \square 6$
One of those sums is greater / not greater than the length of the third side.	It is / is not possible for a triangle to have sides with lengths 2 m, 6 m, and 9 m.



Problem 5 Finding Possible Side Lengths

Got It? A triangle has side lengths of 4 in. and 7 in. What is the range of possible lengths for the third side?

20. Let x = the length of the third side. Use the Triangle Inequality Theorem to write and solve three inequalities.

$$\begin{array}{rcl}
 x + 4 > \square & x + 7 > \square & 7 + 4 > \square \\
 x > \square & x > \square & 11 > \square
 \end{array}$$

21. Underline the correct word to complete each sentence.

Length is always / sometimes / never positive.

The first / second / third inequality pair is invalid in this situation.

22. Write the remaining inequalities as the compound inequality $\square < x < \square$.

23. The third side must be longer than \square in. and shorter than \square in.



Lesson Check • Do you UNDERSTAND?

Error Analysis A friend tells you that she drew a triangle with perimeter 16 and one side of length 8. How do you know she made an error in her drawing?

24. If one side length is 8 and the perimeter is 16, then the sum of the lengths of the two remaining sides must be $16 - 8 = \square$.

25. Underline the correct words or number to complete each sentence.

By the Triangle Inequality Theorem, the sum of the lengths of two sides of a triangle must be equal to / greater than / less than the length of the third side.

By the Triangle Inequality Theorem, the sum of the lengths of the two unknown sides must be equal to / greater than / less than the length 8 / 16.

But 8 is *not* equal to / greater than 8, so there must be an error in the drawing.



Math Success

Check off the vocabulary words that you understand.

exterior angle

comparison property of inequality

Rate how well you can use the *Triangle Inequality Theorem*.

