

# 6-1

## The Polygon Angle-Sum Theorems



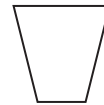
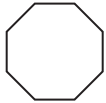
### Vocabulary

#### Review

- Underline the correct word to complete the sentence.

In a *convex* polygon, no point on the lines containing the sides of the polygon is in the interior / exterior of the polygon.

- Cross out the polygon that is NOT *convex*.



#### Vocabulary Builder

**regular polygon** (noun) REG yuh lur PAHL ih gahn

**Definition:** A **regular polygon** is a polygon that is both equilateral and equiangular.

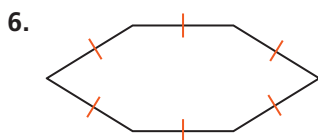
**Example:** An equilateral triangle is a **regular polygon** with three congruent sides and three congruent angles.

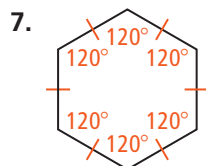
#### Use Your Vocabulary

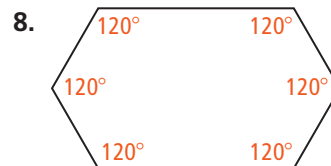
Underline the correct word(s) to complete each sentence.

- The sides of a *regular polygon* are congruent / scalene .
- A right triangle is / is not a *regular polygon*.
- An isosceles triangle is / is not always a *regular polygon*.

Write *equiangular*, *equilateral*, or *regular* to identify each hexagon. Use each word once.







### Theorem 6-1 Polygon Angle-Sum Theorem and Corollary

**Theorem 6-1** The sum of the measures of the interior angles of an  $n$ -gon is  $(n - 2)180$ .

**Corollary** The measure of each interior angle of a regular  $n$ -gon is  $\frac{(n - 2)180}{n}$ .

9. When  $n - 2 = 1$ , the polygon is a(n)   ?  .

10. When  $n - 2 = 2$ , the polygon is a(n)   ?  .



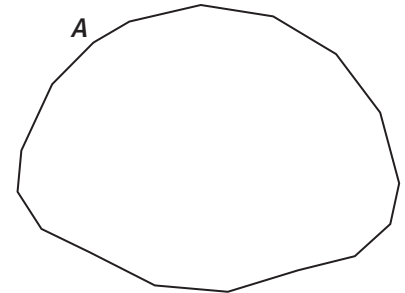
#### Problem 1 Finding a Polygon Angle Sum

**Got It?** What is the sum of the interior angle measures of a 17-gon?

11. Use the justifications below to find the sum.

$$\begin{aligned} \text{sum} &= (\text{   } - 2)180 && \text{Polygon Angle-Sum Theorem} \\ &= (\text{   } - 2)180 && \text{Substitute for } n. \\ &= \text{   } \cdot 180 && \text{Subtract.} \\ &= \text{   } && \text{Simplify.} \end{aligned}$$

12. Draw diagonals from vertex  $A$  to check your answer.



13. The sum of the interior angle measures of a 17-gon is    .



#### Problem 2 Using the Polygon Angle-Sum Theorem

**Got It?** What is the measure of each interior angle in a regular nonagon?

**Underline the correct word or number to complete each sentence.**

14. The interior angles in a regular polygon are congruent / different.

15. A regular nonagon has 7 / 8 / 9 congruent sides.

16. Use the Corollary to the Polygon Angle-Sum Theorem to find the measure of each interior angle in a regular nonagon.

$$\begin{aligned} \text{Measure of an angle} &= \frac{(\text{   } - 2)180}{\text{   }} \\ &= \frac{(\text{   })180}{\text{   }} \\ &= \text{   } \end{aligned}$$

17. The measure of each interior angle in a regular nonagon is    .



### Problem 3 Using the Polygon Angle-Sum Theorem

**Got It?** What is  $m\angle G$  in quadrilateral  $EFGH$ ?

18. Use the Polygon Angle-Sum Theorem to find  $m\angle G$  for  $n = 4$ .

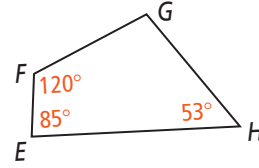
$$m\angle E + m\angle F + m\angle G + m\angle H = (n - 2)180$$

$$m\angle E + m\angle F + m\angle G + m\angle H = (\square - 2)180$$

$$\square + \square + \square + \square = \square \cdot 180$$

$$m\angle G + \square = \square$$

$$m\angle G = \square$$



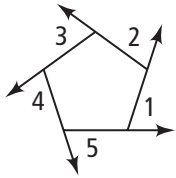
19.  $m\angle G$  in quadrilateral  $EFGH$  is  $\square$ .

take note

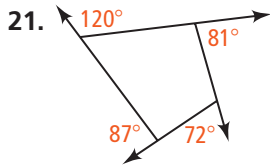
### Theorem 6-2 Polygon Exterior Angle-Sum Theorem

The sum of the measures of the exterior angles of a polygon, one at each vertex, is 360.

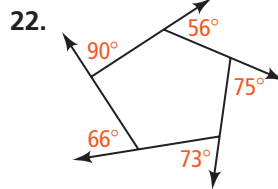
20. In the pentagon below,  $m\angle 1 + m\angle 2 + m\angle 3 + m\angle 4 + m\angle 5 = \square$ .



Use the Polygon Exterior Angle-Sum Theorem to find each measure.



$$120 + 81 + \square + 87 = 360$$



$$90 + \square + 75 + 73 + 66 = \square$$



### Problem 4 Finding an Exterior Angle Measure

**Got It?** What is the measure of an exterior angle of a regular nonagon?

Underline the correct number or word to complete each sentence.

23. Since the nonagon is regular, its interior angles are congruent / right .

24. The exterior angles are complements / supplements of the interior angles.

25. Since the nonagon is regular, its exterior angles are congruent / right .

26. The sum of the measures of the exterior angles of a polygon is 180 / 360 .

27. A regular nonagon has 7 / 9 / 12 sides.

28. What is the measure of an exterior angle of a regular nonagon? Explain.

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### Lesson Check • Do you UNDERSTAND?

**Error Analysis** Your friend says that she measured an interior angle of a regular polygon as 130. Explain why this result is impossible.

29. Use indirect reasoning to find a contradiction.

Assume temporarily that a regular  $n$ -gon has a  $130^\circ$  interior angle.

angle sum =   $\cdot n$       A regular  $n$ -gon has  $n$  congruent angles.

angle sum =  $(\text{input})180$       Polygon Angle-Sum Theorem

=  $(\text{input})180$       Use the Transitive Property of Equality.

=  -       Use the Distributive Property.

=       Subtract  $180n$  from each side.

$n = \text{input}$       Divide each side by  $-50$ .

$n \neq \text{input}$       The number of sides in a polygon is a whole number  $\geq 3$ .

30. Explain why your friend's result is impossible.

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### Math Success

Check off the vocabulary words that you understand.

equilateral polygon       equiangular polygon       regular polygon

Rate how well you can *find angle measures of polygons*.

