9-6 Compositions of Reflections

Vocabulary

Review

Write T for *true* or F for *false*.

_____ **1.** A *reflection* flips a figure across a line of *reflection*.

_____ **2.** A *reflection* turns a figure about a point.

_____ **3.** A *reflection* preimage and image are congruent.

_____ **4.** The orientation of a figure reverses after a *reflection*.

_____ **5.** A line of *reflection* is either horizontal or vertical.

Vocabulary Builder

composition (noun) kahm puh ZISH un

Other Word Forms: compose (verb), composite (adjective), composite (noun)

Definition: A **composition** combines parts.

Math Usage: A **composition** of transformations combines two or more transformations in a given order.

Use Your Vocabulary

Complete each statement with the appropriate word from the list. Use each word only once.

reflections rotation symmetry

6. A *composition* of reflections has at least one line of _?_. _____

7. You can map any congruent figure onto another using a *composition* of _?_. _____

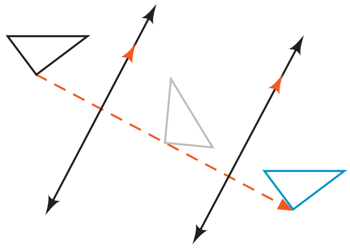
8. A *composition* of rotations is always a _?_. _____

take note — Theorem 9-1 and Theorem 9-2

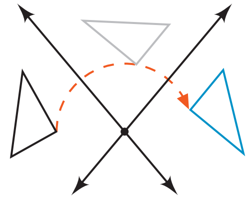
Theorem 9-1 A translation or rotation is a composition of two reflections.

Theorem 9-2

A composition of reflections across parallel lines is a translation.

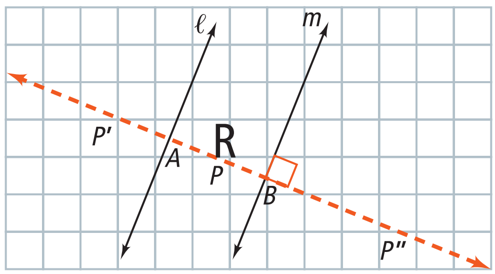


A composition of reflections across two intersecting lines is a rotation.



Problem 1 Composing Reflections Across Parallel Lines

Got It? **Lines ℓ and m are parallel. R is between ℓ and m. What is the image of R reflected first across line ℓ and then across line m? What are the direction and distance of the resulting translation?**



9. The diagram shows a dashed line perpendicular to ℓ and m that intersects ℓ at point A, m at point B, and R only at point P. Complete each step to show the composition of the reflections.

Step 1 Reflect R across line ℓ. Point P' should correspond to point P.

Step 2 Reflect the image across line m. Point P'' should correspond to point P'.

10. Underline the correct word to complete each sentence.

The translation is to the right / left along the dashed line.

The direction of the translation is parallel / perpendicular to lines ℓ and m.

11. Use the justifications at the right to find the distance PP'' of the resulting translation.

PP''	$= \square + BP''$	Segment Addition Postulate
	$= \square + BP'$	Definition of reflection across line m
	$= \square + (BP + PA + AP')$	Segment Addition Postulate
	$= \square + BP + 2PA$	Definition of reflection across line ℓ
	$= \square \cdot BP + 2PA$	Simplify.
	$= \square \cdot (BP + PA)$	Use the Distributive Property.
	$= \square \cdot \square$	Segment Addition Postulate

12. The resulting translation moved R a distance of ______ .

take note — Theorem 9–3 Fundamental Theorem of Isometries

In a plane, one of two congruent figures can be mapped onto the other by a composition of at most three reflections.

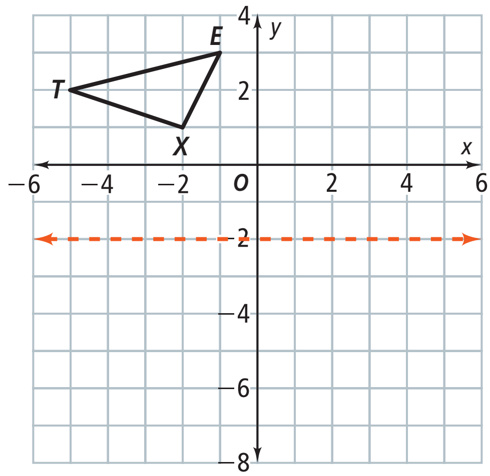
13. Underline the correct word to complete the sentence.

If two congruent figures in a plane have opposite orientations, an even / odd number of reflections maps one figure onto the other.

Problem 3 Finding a Glide Reflection Image

Got It? **What is the image of $\triangle TEX$ for a glide reflection where the translation is $(x, y) \rightarrow (x + 1, y)$ and the line of reflection is $y = -2$?**

Use the coordinate plane at the right for Exercises 14–17.



14. Find the vertices of the translation image. Then graph the translation image.

$T(-5, 2) \rightarrow (-5 +$ ____ , ____ $) = ($ ____ , ____ $)$

$E(-1, 3) \rightarrow (-1 +$ ____ , ____ $) = ($ ____ , ____ $)$

$X(-2, 1) \rightarrow (-2 +$ ____ , ____ $) = ($ ____ , ____ $)$

15. In a reflection across a horizontal line, only the ____ -coordinate changes.

16. Find the vertices of the triangle you graphed in Exercise 14 after reflection across the line $y = -2$.

(____ , ____) $\rightarrow T'($ ____ , ____ $)$

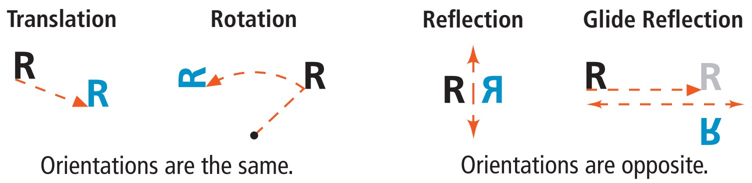
(____ , ____) $\rightarrow E'($ ____ , ____ $)$

(____ , ____) $\rightarrow X'($ ____ , ____ $)$

17. The image of $\triangle TEX$ for the given glide reflection is the triangle with vertices $T'($ ____ , ____ $)$, $E'($ ____ , ____ $)$, and $X'($ ____ , ____ $)$. Graph $\triangle T'E'X'$.

take note — Theorem 9–4 Isometry Classification Theorem

There are only four isometries.



Problem 4 Classifying Isometries

Got It? **Each figure is an isometry image of the figure at the right. Are the orientations of the preimage and image the *same* or *opposite*? What type of isometry maps the preimage to the image?**

A. **B.** **C.**

Choose the correct words from the list to complete each sentence.

opposite
same
translation
rotation
reflection
glide reflection

18. Image A has the _?_ orientation and is a _?_.

______________ ______________

19. Image B has the _?_ orientation and is a _?_.

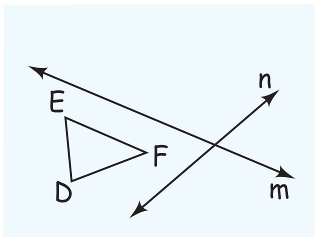
______________ ______________

20. Image C has the _?_ orientation and is a _?_.

______________ ______________

Lesson Check • Do you UNDERSTAND?

Error Analysis **You reflect $\triangle DEF$ first across line *m* and then across line *n*. Your friend says you can get the same result by reflecting $\triangle DEF$ first across line *n* and then across line *m*. Explain your friend's error.**



21. Place a ✓ in the box if the response is correct. Place an ✗ if it is incorrect.

☐ Lines *m* and *n* are perpendicular.

☐ A clockwise or counterclockwise rotation has the same image.

22. Explain your friend's error.

____

Math Success

Check off the vocabulary words that you understand.

☐ composition of reflections ☐ glide reflection ☐ isometry

Rate how well you can *find compositions of reflections.*

Need to review | 0 2 4 6 8 10 | Now I get it!