Circles Unit

12-1 Tangent Lines

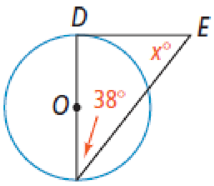
Vocabulary

-Tangent, Point of Tangency

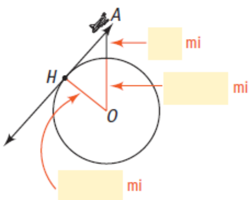
Tangent Lines

Theorem 12-1 If a line is tangent to a circle, then the line is perpendicular to the radius at the point of tangency. The converse is also true.

Got It? $\overline{ED}$ is tangent to $\odot O$. What is the value of x?



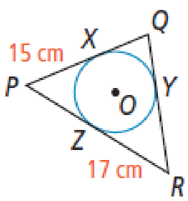
Got It? What is the distance to the horizon that a person can see on a clear day from an airplane 2 mi above Earth? Earth's radius is about 4000 mi.



Tangent Segments that Share an Endpoint

Theorem 12-3 If two tangent segments to a circle share a common endpoint outside the circle, then the two segments are congruent.

Got It? $\odot O$ is inscribed in $\triangle PQR$, which has a perimeter of 88 cm. What is the length of $\overline{QY}$?

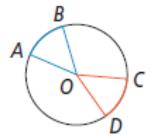


12-2 Chords and Arcs

Vocabulary

-Chord, Arc

Chords and Arcs

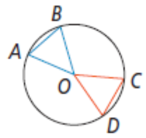


Theorem 12-4 Within a circle or in congruent circles, congruent central angles have congruent arcs.

4. If $\angle AOB \cong$ ______, then $\overset{\frown}{AB} \cong \overset{\frown}{CD}$.

Converse Within a circle or in congruent circles, congruent arcs have congruent central angles.

5. If $\overset{\frown}{AB} \cong \overset{\frown}{CD}$, then $\angle AOB \cong$ ______.

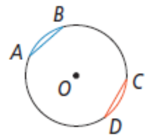


Theorem 12-5 Within a circle or in congruent circles, congruent central angles have congruent chords.

6. If $\angle AOB \cong \angle COD$, then $\overline{AB} \cong$ ______.

Converse Within a circle or in congruent circles, congruent chords have congruent central angles.

7. If $\overline{AB} \cong \overline{CD}$, then ______ $\cong \angle COD$.



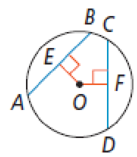
Theorem 12-6 Within a circle or in congruent circles, congruent chords have congruent arcs.

8. If $\overline{AB} \cong$ ______, then $\overset{\frown}{AB} \cong \overset{\frown}{CD}$.

Converse Within a circle or in congruent circles, congruent arcs have congruent chords.

9. If $\overset{\frown}{AB} \cong$ ______, then $\overline{AB} \cong \overline{CD}$.

Theorem 12-7 Within a circle or in congruent circles, chords equidistant from the center or centers are congruent.

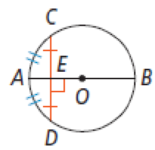


Converse Within a circle or in congruent circles, congruent chords are equidistant from the center (or centers).

11. If $OE = OF$, then $\overline{AB} \cong$ ______.

12. If $\overline{AB} \cong$ ______, then $OE =$ ______.

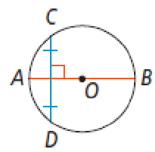
Theorem 12-8 In a circle, if a diameter is perpendicular to a chord, then it bisects the chord and its arc.



13. If $\overline{AB}$ is a diameter and $\overline{AB} \perp \overline{CD}$, then $\overline{CE} \cong$ ______ and $\overset{\frown}{CA} \cong$ ______.

Theorem 12-9 In a circle, if a diameter bisects a chord (that is not a diameter), then it is perpendicular to the chord.

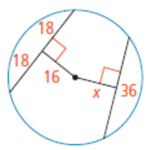
14. If $\overline{AB}$ is a diameter and $\overline{CE} \cong \overline{ED}$, then $\overline{AB} \perp$ ______.



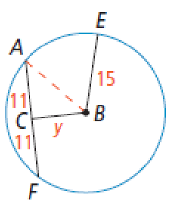
Theorem 12-10 In a circle, the perpendicular bisector of a chord contains the center of the circle.

15. If $\overline{AB}$ is the perpendicular bisector of chord $\overline{CD}$, then ______ contains the center of $\odot O$.

Got It? What is the value of x? Justify your answer.



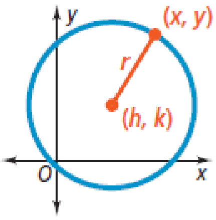
Got It? Reasoning In finding y, how does the auxiliary $\overline{BA}$ make the problem simpler to solve?



12-5 Circles in the Coordinate Plane

Vocabulary

-Standard Form Equation of a Circle



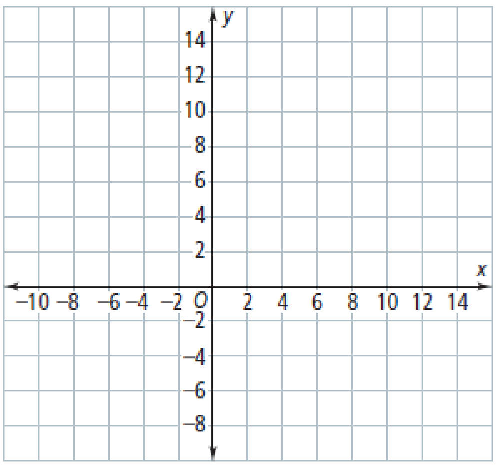
Standard Form Equation of a Circle

An equation of a circle with center (h, k) and radius r is $(x - h)^2 + (y - k)^2 = r^2$.

12. The equation of a circle with center $(-1, 0)$ and $r = 6$ is $(x + 1)^2 + (y - 0)^2 = 6^2$.

Got It? What is the standard equation of the circle with center (3, 5) and radius 6?

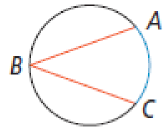
Got It? What is the center and radius of the circle with equation $(x - 2)^2 + (y - 3)^2 = 100$? Graph the circle.



12-3 Inscribed Angles

Vocabulary

-Inscribed Angle, Intercepted Arc



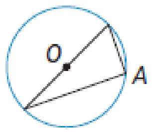
Inscribed Angle Theorem

The measure of an inscribed angle is half the measure of its intercepted arc.

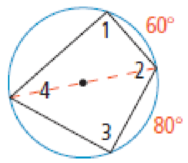
$m\angle B = \frac{1}{2} m\widehat{AC}$

Use this formula when the vertex of the angle is ON the circle!
SPECIAL NOTE: A circle is divided into 360 degrees!

Got It? In ⊙O, what is $m\angle A$?

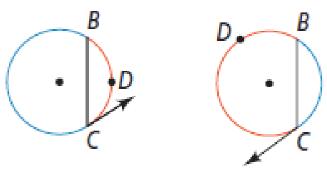


Got It? In the diagram at the right, what is the measure of each numbered angle?

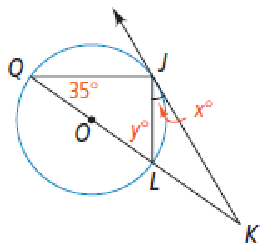


The measure of an angle formed by a tangent and a chord is half the measure of the intercepted arc.

$m\angle C = \frac{1}{2} m\widehat{BDC}$



Got It? In the diagram at the right, $\overline{KJ}$ is tangent to ⊙O. What are the values of x and y?



12-4 Angle Measurements and Segment Lengths

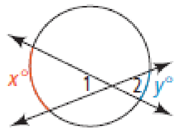
Vocabulary

-Secant

Angles Formed When Secants Intersect INSIDE Circle

Theorem 12-13 The measure of an angle formed by two lines that intersect inside a circle is half the sum of the measures of the intercepted arcs.

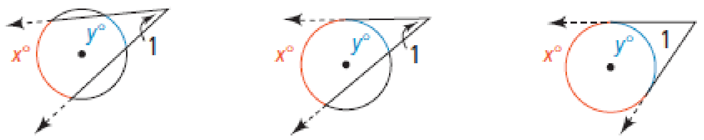
$m\angle 1 = \frac{1}{2}(x + y)$



Angles Formed When Secants Intersect OUTSIDE Circle

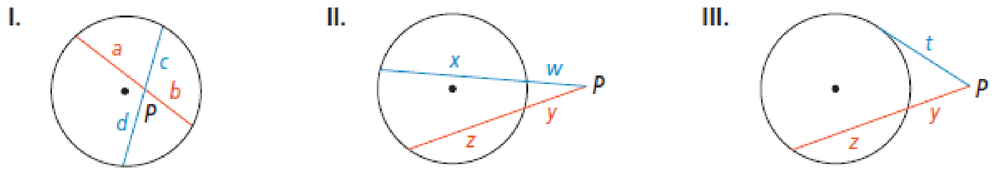
Theorem 12-14 The measure of an angle formed by two lines that intersect outside a circle is half the difference of the measures of the intercepted arcs.

$m\angle 1 = \frac{1}{2}(x - y)$



Segments and Circles

Theorem 12-15 For a given point and circle, the product of the lengths of the two segments from the point to the circle is constant along any line through the point and the circle.



Complete each case of Theorem 12-15.

14. Case I $a \cdot b = c \cdot$ d

15. Case II $(w + x)\,w = ($ y $+ z)$ y

16. Case III $(y + z)$ y $= t^2$

Find the values of all the missing variables at the right.